

Graphics

1 - DFT Math

$$x = \{1,0,1,0,1,1,0,1,0,1,0,0\}$$

$$f_k = \sum_{n=0}^{11} x_n e^{\frac{-i2\pi}{12}nk}$$

f_k **Fourier Coefficient:** 7 complex numbers

$\sum_{n=0}^{11}$ **Summation Function:** add up all the numbers

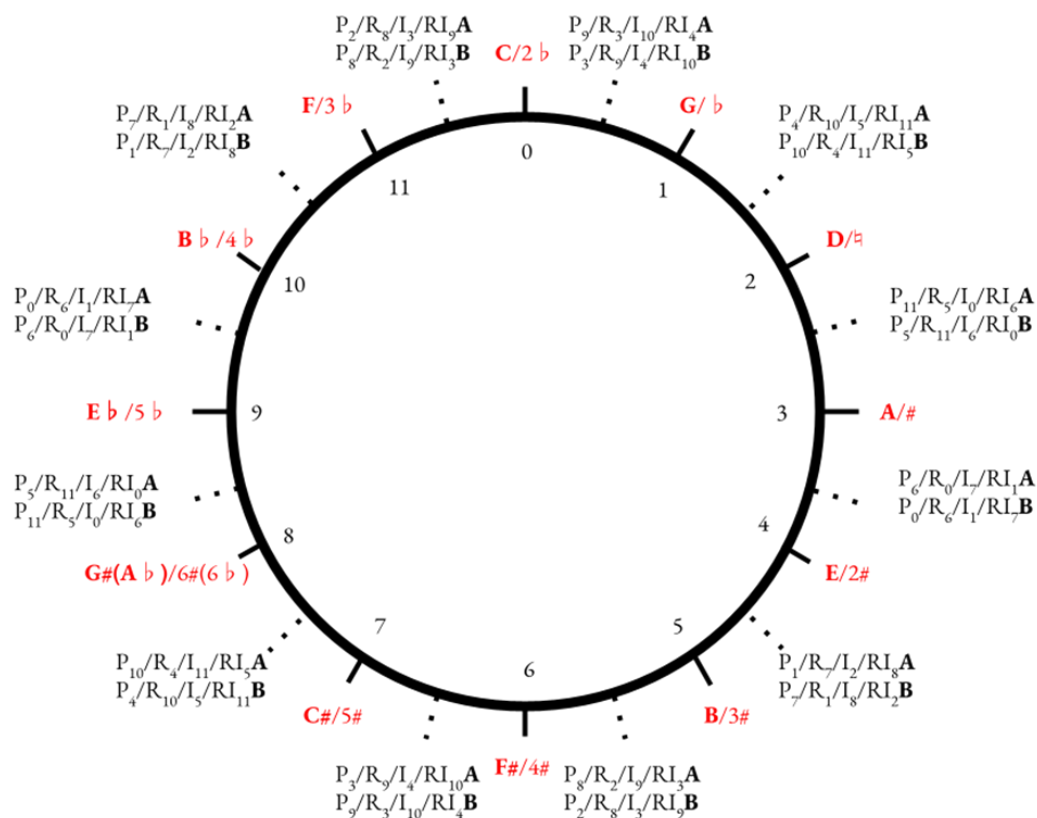
x_n **Pitch Weight:** the number at the n th location at our characteristic function

$e^{\frac{-i2\pi}{12}nk}$ **Mathematical shorthand** that fits the function to various periodicities

Euler's formula: $e^{ix} = \cos x + i \sin x$

$$e^{\frac{-i2\pi}{12}nk} = \cos \frac{-2\pi nk}{12} + i \sin \frac{-2\pi nk}{12}$$

2 - Ph₅ Circle



3 – Row and $|f_n|^2$ of partitions

The image shows a musical staff with a treble clef and a key signature of one flat (B-flat). The row consists of 12 notes: B-flat, C, D, E-flat, F, G, A-flat, B-flat, C, D, E-flat, F. The notes are grouped into partitions with brackets and set class labels above or below them:

- Trichord first: (015) [B-flat, C, D]
- Trichord last: (037) [E-flat, F, G]
- Trichords middle: (026) [A-flat, B-flat, C]
- Tetrachord first: (0148) [B-flat, C, D, E-flat]
- Tetrachord last: (0358) [A-flat, B-flat, C, D]
- Tetrachord middle: (0147) [E-flat, F, G, A-flat]
- Pentachords: (01358) [B-flat, C, D, E-flat, F]
- Hexachords: (014579) [B-flat, C, D, E-flat, F, G]

Segment	Set class	$ f_1 ^2$	$ f_2 ^2$	$ f_3 ^2$	$ f_4 ^2$	$ f_5 ^2$	$ f_6 ^2$
Trichord first	(015)	2	4	5	0	2	1
Trichord last	(037)	0.27	1	5	3	3.73	1
Trichords middle	(026)	1	3	1	3	1	9
Tetrachord first	(0148)	0.27	3	8	1	3.73	0
Tetrachord last	(0358)	0.54	0	4	4	7.46	0
Tetrachord middle	(0147)	1	3	4	7	1	0
Pentachords	(01358)	1.54	1	5	1	8.46	1
Hexachords	(014579)	0.27	1	8	3	3.73	4

Scores

8

N. 4 - LINEE

Tranquillamente mosso (♩ = 132)

piano

mp; sost.

(poco)

mp; intenso

piano

più p

40 secondi

S. 4059 Z.

Affettuoso; cullante (♩=120)

p *mp; espr.* *pp*

con Sord.

mp; espr. *più p*

ppp *(più sottovoce)* *7 m.s.*

(sempre espr.)

sempre più sottovoce *ppp*

Molto lento; fantastico (♩ = 40)

con grande espress.

pp; sost.

espr.

non arp.

con la massima espressione

(sostenutiss.)

espr.

ppp

ppp

m. s.

1 min. 20 secondi
 Montréal - New York - Mexico City
 (Agosto - Settembre 1952)
 [ritoccato nell'estate 1953]

[S. 4959 Z.]

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